Model Development for Quantifying and Predicting Rainfall-Induced Erosion in Tropics Region Using Dimensional Equation Approach

Aboshio Nehemiah[1*], Mohammed M. Maina[2], Nura J. Shanono[2], Kayong Emmanuel Anthony[1], Bature James Boman[1] and Nale Biki Yusuf[3]

[1]Department of Agricultural and Bio Environmental Engineering, School of Agricultural Technology, Nuhu Bamalli Polytechnic Zaria, Kaduna State Nigeria

[2]Department of Agricultural and Environmental Resources Engineering, Faculty of Engineering, Bayero University, Kano, Nigeria

[3]Department of General Studies, School of Agricultural Technology, Nuhu Bamalli Polytechnic Zaria, Kaduna State Nigeria

*Corresponding Author:- Email: naboshio@yahoo.co.uk Tel: +2348037313207

Abstract

Rainfall-induced soil erosion was developed in Samaru-Kataf, zangon-Kataf local government area Kaduna state, Nigeria. Data was collected in 2018, 2019 and 2020 and was used to develop a dimensionless model for predicting Rainfall induced erosion using the dimensional analyses approach consideringHydraulic shear stress, Soil erodibility factor K, Soil infiltration rate, Depth of tilt, Raindrop size, Rainfall intensity, Runoff velocity, Slope length, Vegetation density on the experimental field plot as the building components. The component equation derived from the regression's equation was used to develop the model equation. The R^2 value of 0.635 indicates a high degree of agreement between the model output and the field measured soil erosion data.A slope of 0.014, y-intercept of 0.0,index of agreement (D) of 0.069, Nash- Sutcliffe Efficiency (NSE)0.035indicate that the model significantly reproduces the magnitudes of measured data, the correlation coefficient of 0.846indicates that a strong positive linear relationship exists. Generally, the analysis demonstrated a good predictive capacity of the model. The sensitivity analysis revealed that the model is most sensitive to changes in hydraulic shear stress this could be attributed to its role in detachment and transportation of the soil particles. It is recommended for use in tropic regions because variables are easily determined which makes it better and easily adaptable. Likewise it is capable of measuring soil loss from sheet/rill and gully erosion. Similarly, the model is capable of measuring weekly, monthly and yearly soil loss of an area and it accounts for soil deposition.

***Keywords**: Soil Erosion, sedimentation, Prediction, Rainfall Induced, Dimensional Analyses.*

INTRODUCTION

The livelihoods of humankind are vastly influenced by soil erosion. The study conducted by Devatha *et al*., (2015) specifically identified some of the negative effects of soil erosion which include, but are not limited to pollution of nearby water bodies and wetland, a decline in cropland productivity, and disturbance of delicate aquatic ecosystems. In addition, most people in the world rely on soil resources as their main livelihood source (Benchettouh *et al*., 2017).

Molla and Sisheber, (2017) reported that among the major causes of soil erosion is rainwater, which breaks up the soil, dislodges it from its surroundings, and washes it subsequently away as runoff. The prevalent of this environmental problem is also largely due to man-induced activities such as land management and agricultural practices which contribute in making the problem of soil erosion very

difficult to deal with (Obeta & Adewumi, 2013). Soil erosion is a worldwide environmental problem that reduces the productivity of all-natural ecosystems and agricultures, which threatens the lives of most smallholder farmers (Flanagan *et al*., 2012). If left unchecked, as Avwunudiogba & Hudson, (2014) claimed, this problem will lead to food insecurity, without healthy soils, farmers wouldn't be able to provide us with feed, fiber, and food.

Over the years, several soil erosion models have been developed by many researchers to estimate soil erosion, sediment yield, and deposition on a hill slope and small watersheds under a wide range of conditions to control the soil erosion (Benavidez *et al*., 2018; Chandramohan *et al*., 2015; and Igwe *et al*., 2017). Likewise, Merritt *et al*., (2003) classified soil erosion models into three categories depending on the physical processes simulated by the model, the model algorithms describing the processes, and the data dependence of the model: empirical or statistical, conceptual, and physical base models.

The limitations of the soil erosion models impact negatively on the management of soil loss associated with runoff and erosion on agricultural land. As Kinnell, (2010) reported, the Revised Universal Soil Loss Equation (RUSLE) and Water Erosion Prediction Project (WEPP) are the widely used erosion models in the world and Nigeria in particular. First, Igwe *et al*., (2017; and Suresh, (2002) reported that errors in the selection of the appropriate value of the different factors get introduced in the computation of soil loss in the RUSLE equation. Particularly, care should be taken for choosing those values, which are based on crop concept because a mistake/ error in chosen one or more factors may lead to accumulation of error which tend to give the wrong estimate of soil loss. Basically, RUSLE equations are developed based on annual soil loss data; hence their applications are limited to estimating only the average annual soil loss of a given area (Aksoy & Kavvas, 2005; Benavidez *et al*., 2018; Kinnell, 2010; and Suresh, 2002). RUSLE computes less value than the measured, especially when rainfall occurs at a high rate (Suresh, 2002) and the significance of slope steepness in the areas of intense rainfall is ignored. Likewise, the computation of rainfall erosion index based on the drop-size distribution of rain is not available and runoff amount and its velocity is known to significantly affect the soil loss is not taken into account (Suresh, 2002). Also, Benchettouh *et al*., (2017); Morgan *et al*., (1998); and Wischmeier & Smith, (1978) reported that RUSLE and WEPP are employed for assessing the sheet and rill erosions and not used for predicting the gully erosion. The gully erosion, caused by concentrated water flow is not counted by the equations and yet can produce a greater volume of eroded soil (Benavidez *et al*., 2018). Furthermore, Morgan *et al*., (1998) reported that these equations estimate only soil loss, but not the soil deposition. The deposition of sediment at bottom of the channel is being less than the total soil loss taking place from the entire watershed. The primary limitation of WEPP is that it can only model a small area (< 2.6 km^2). In larger watersheds, users will often need to use advanced modelling skills to incorporate climate variability within the watershed and to combine results, runs on numerous small sub-watersheds within the study area.

Regional erosion models EUROSEN Morgan *et al*., (1998) developed to provide alternatives to the European community and Soil Loss Estimator for Southern Africa SLEMSA Elwell, (1978). Similarly, other erosion models like SHESED (Wicks *et al*., 1992), LISEM De Roo *et al*., (1996), WEHY Kavvas *et al*., (2004) and Xiaoming (2010) developed to estimate soil loss but their applicability is limited to

an area with similar soil and weather conditions.

Hence, it is precisely in this context that the research aims at formulating a simple, easy to use model, compatible with the tropical region. Therefore, there is a need to develop a rainfall-induced erosion model because knowing and understanding the difficulties and limitations encountered in estimation sol loss using RUSLE and WEPP put the researcher in the best position to offer some alternative to the problem.

The study will be of benefit to the Government at the state and federal level and tertiary institutions in the locality. Moreover, it will provide soil conservationists, Engineers, farmers, researchers and other land users with the tools they need to examine the impact of various management strategies on soil loss and plan for the optimal use of the land.

The rainfall-induced erosion model will not only minimize error in assigning/selecting the appropriate values of the different factors associated with RUSLE and WEPP but also compute both sheet/rill and gully erosion. At the same time, the model is capable of measuring weekly, monthly, and yearly soil loss of an area and account for deposition of sediment.

METHODOLOGY

Study Area

The research was conducted at the experimental farm, School of Agriculture Technology, Nuhu Bamalli Polytechnic Zaria, Samara-Kataf Campus along Kachia-Kafanchan Road *N90°46`35" E008 22`46"*. Samaru- Kataf is located in the Southern part of Kaduna State, Zangon Kataf LGA, approximately 200km from the state capital as shown in the map presented in Figure1. It has two predominant seasons, dry and wet. The wet season starts early April and extends to October during which many agricultural activities are carried out while the dry season commences late October and persists until late March.

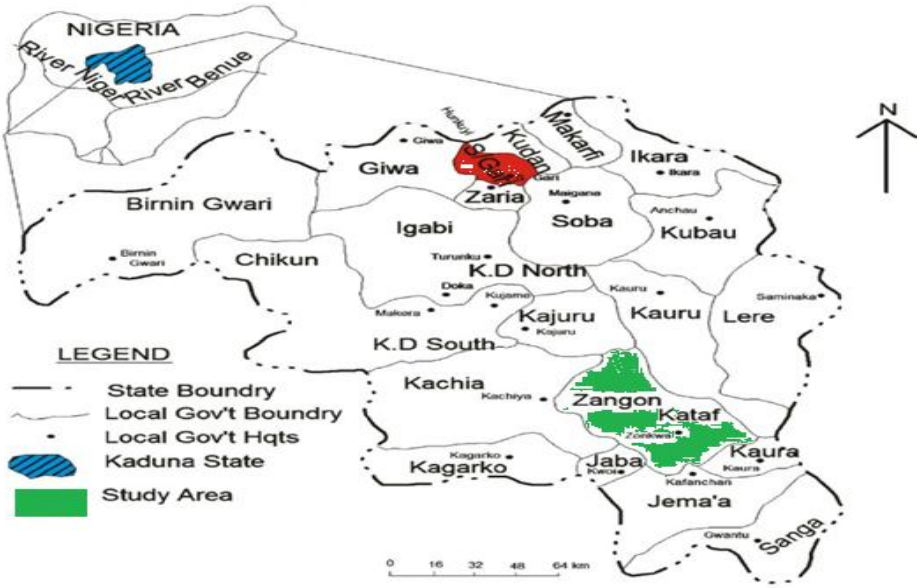

Figure1: Map of Kaduna State showing Zangon Kataf L.G.A
Source: Obeta & Adewumi, 2013

Model Development

The rainfall-induced erosion model was developed using dimensional analysis following the concept of Buckingham π-theorem considering the variables in Table 1.

Buckingham Pi Theorem

Buckingham pi theorem is a system of forming dimensionless groups for an equation involving more than four variables. If an equation involving N variables is dimensionally sourced, it can be reduced to a relationship among N-r independent dimensionless products, where r is the minimum number of reference dimensions (MLT) required to describe the variable. The dimensionless products are referred to as pi terms.

Table 1; Variables and their corresponding dimension

Variable	Symbols	Units	Dimensional symbol (MLT)
Soil erosion	E	$Kg/m^2/yr$	$ML^{-2}T^{-1}$
Soil infiltration rate	I	mm/hr	LT^{-1}
Soil erodibility factor	K	$Kg.hr/Nm^2$	$L^{-3}T^3$
Hydraulic shear stress	□	N/m^2	$ML^{-1}T^{-2}$
Depth of tilt	D_t	m	L
Raindrop size	R_s	m	L
Rainfall intensity	R_i	mm/hr	$LT^{—1}$
Runoff velocity	R_v	m/s	$LT^{—1}$
Slope length	S_l	m	L
Vegetation density	V_d	$Number/m^2$	L^{-2}

The variables in Table1. were picked from soil erosion models (Avwunudiogba & Hudson, 2014; Flanagan *et al*., 1995; Igwe *et al*., 2017; and Morgan *et al*., 1998).

Determination of Pi Terms

Soil erosion E, depend on Soil infiltration rate I, Soil erodibility factor K, Hydraulic shear stress □, Depth of tilt D_t, Raindrop sizes R_s, Rainfall intensity R_i, Runoff velocity R_v, Slope length S_l and Vegetation density V_d,

Step1. Here, E is a function of I, K, □, D_t, R_s, R_i, R_V, S_l V_d

$$E = F(I, K, □, D_t, R_s, R_i, R_V, S_l V_d) \quad (1)$$

Step2. The functional relationship between the dependent and independent variables were written as:

$$F(I, K, □, D_t, R_s, R_i, R_V, S_l V_d) = 0 \quad (2)$$

Step3. Total number of variables N in Table 2.1 = 10

Step4. The dimension of the variables

Soil erosion $E = ML^{-2}T^{-1}$

Soil infiltration rate $I = LT^{-1}$

Soil erodibility factor $K = L^{-3}T^3$

Density of vegetation $V_d = L^{-2}$

Depth of tilt $D_t = L$

Raindrop sizes $R_s = L$

Rainfall intensity $R_i = LT^{-1}$

Runoff velocity $Rv = LT^{-1}$

Slope length $S_l = L$

Hydraulic shear stress □ $= ML^{-1}T^{-2}$

Step 5. Number of fundamental dimensions (r),

$$MLT = 3 \quad (3)$$

Step6. Number of pi terms required= N-r,

$$10 - 3 = 7 \text{ pi terms} \quad (4)$$

$F_1(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7)$

Step7. Selection of repeating variables, The variable in Table 1. were grouped

according to their properties into; Geometric properties (D_t , R_s , S_l), Flow properties (I, R_i, R_V) and Material properties (K, □,V_d , E).

One repeating variable was selected from each group. The repeating variables are independent of the other variables.

The tree repeating variable selected plus one variable were arranged as follows for π_1 to π_7.

$$\pi_1 = S_l^{a_1} I^{b_1 c_1} E \quad (5)$$
$$\pi_2 = S_l^{a_2} I^{b_2 c_2} K \quad (6)$$
$$\pi_3 = S_l^{a_3} I^{b_3 c_3} V_d \quad (7)$$
$$\pi_4 = S_l^{a_4} I^{b_4 c_4} D_t \quad (8)$$
$$\pi_5 = S_l^{a_5} I^{b_5 c_5} R_s \quad (9)$$
$$\pi_6 = S_l^{a_6} I^{b_6 c_6} R_i \quad (10)$$
$$\pi_7 = S_l^{a_7} I^{b_7 c_7} R_v \quad (11)$$

Step 8.

Determination of π_1

The dimension of the variables in step 4 was substituted on both sides of (Eqn 5.)

$$M^0L^0T^0 = (L)^{a_1}(LT^{-1})^{b_1}(ML^{-1}T^{-2})^{c_1}ML^{-2}T^{-1} \quad (12)$$

$$M^0L^0T^0 = M^{c_1+1}L^{a_1+b_1-c_1-2}T^{-b_1-2c_1-1} \quad (13)$$

The power of M on both sides of Eqn (13) was compared to give,

$$0 = c_1 + 1 \quad (14)$$
$$c_1 = -1$$

The power of L on both sides of Eqn (13) was compared to give,

$$0 = a_1 + b_1 - c_1 - 2 \quad (15)$$
$$0 = a_1 + b_1 - (-1) - 2$$
$$0 = a_1 + b_1 + 1 - 2$$
$$0 = a_1 + b_1 - 1$$
$$a_1 + b_1 = 1$$
$$a_1 = 0$$

The power of T on both sides of Eqn (13) was compared to give,

$$0 = -b_1 + 1 \quad (15)$$
$$b_1 = 1$$

Substituting the values of a_1, b_1, c_1 in Eqns (5) yield:

$$\pi_1 = S_l^0 I^{1-1} E$$
$$\pi_1 = \frac{IE}{} \quad (16)$$

Determination of π_2

The dimensions of the variables in step 4 were substituted on both sides of (Eqn 6)

$$M^0L^0T^0 = (L)^{a_2}(LT^{-1})^{b_2}(ML^{-1}T^{-2})^{C_2}L^{-3}T^3 \quad (17)$$

$$M^0L^0T^0 = M^{c_2}L^{a_2+b_2-c_2-3}T^{-b_2-2c_2+3} \quad (18)$$

The power of M on both sides of Eqn (18) was compared to give,

$$0 = c_2 \quad (19)$$
$$c_2 = 0$$

The power of L on both sides of Eqn (18) was compared to give,

$$0 = a_2 + b_2 - c_2 - 3 \quad (20)$$
$$0 = a_2 + b_2 - (0) - 3$$
$$0 = a_2 + b_2 - 3$$
$$a_2 + b_2 = 3$$
$$a_2 = 0$$

The power of T on both sides of Eqn (18) was compared to give,

$$0 = -b_2 - 2c_2 + 3 \quad (21)$$
$$0 = -b_2 - 2(0) + 3$$
$$0 = -b_2 + 3$$
$$b_2 = 3$$

Substituting the values of a_2, b_2, c_2 in Eqns (6) yield:

$$\pi_2 = S_l^0 I^{30} K$$
$$\pi_2 = I^3 K \quad (22)$$

The above procedure was used to develop four other π-terms.

Summary of the Pi terms Developed

$$\pi_1 = \frac{IE}{}, \pi_2 = I^3K, \pi_3 = S_l^2 V_d, \quad \pi_4 = \frac{D_t}{S_l},$$
$$\pi_5 = \frac{R_s}{S_l}, \quad \pi_6 = \frac{R_i}{I}, \quad \pi_7 = \frac{R_v}{I}$$

Combination of Pi Terms

The number of pi terms developed was further reduced by multiplying:

$$\pi_{1n} = \pi_1 \times \pi_4 \times \pi_7$$
$$\pi_{1n} = \frac{IE}{} \times \frac{D_t}{S_l} \times \frac{R_v}{I} = \frac{ED_tR_v}{S_l} \quad (23)$$
$$\pi_{2n} = \pi_2 \times \pi_5$$

$$\pi_{2n} = I^3K \times \frac{R_s}{S_l} = \frac{I^3KR_s}{S_l} \quad (24)$$

$$\pi_{3n} = \pi_3 \times \pi_6$$

$$\pi_{3n} = S_l^2V_d \times \frac{R_i}{I} = \frac{S_l^2R_iV_d}{I} \quad (25)$$

The pi terms were multiplied to minimize the number of dimensionless terms to handle. The pi terms multiplied satisfied the requirement for product rule given by Buckingham.

The pi terms combined must be from different groups of properties.

$\pi_{1n} = \frac{ED_tR_v}{S_l}$, $\pi_{2n} = \frac{I^3KR_s}{S_l}$, $\pi_{3n} = \frac{S_l^2R_iV_d}{I}$

Many other transformations are possible. The requirements are that the pie terms be dimensionless and independent.

Determination of Parameters for Variables Used in Model Development

Soil Infiltration Rate

A double-ring infiltrometer was used to determine the soil infiltration rate.

Soil Erodibility K

Nomograph was used to get the soil Erodibility;

Hydraulic Shear Stress τ

The hydraulic shear stress formula was used to determine the Hydraulic shear stress τ

$$\tau = \gamma RS$$

Where;

τ is the Hydraulic shear stress (Nm^{-2})

γ is the Specific (unit) weight of water which is given as 9.804 (Nm^{-3})

R is the Hydraulic radius (m)

S is the Slope

Depth of Tilt

The field was tilted manually to an average depth of (10cm) using a big hoe (garma).

Rain Drop Diameter

The flour pellet method was used to determine the raindrop diameter.

Rainfall Intensity R_i

The rain gauge was used to determine the rainfall intensities:

Runoff Velocity

Runoff velocity was calculated using the velocity area method (float gauge) with measuring tape, blue coloured dye, and stop-watch (timer).

Vegetation Density

The quadrate method was used to determine the density of vegetation

Data Collection

The Cluster random sampling method was used to eliminate bias in sampling data. Samples were taken every week, depending on rainfall for seven months starting from April to October for three years (2018-2020).

The mean of the data obtained in a week formed a cluster sample. The Cluster sample was randomly selected every month to be used in developing the model while the remaining three cluster samples were used for validating the model.

Model Performance Evaluation

The evaluation of rainfall induces erosion model behavior and performance was made through comparisons of simulated and observed variables. How well a model simulation fits the observed data was evaluated by using statistical measures commonly referred to as efficiency criteria, efficiency measures. Efficiency criteria are derived from the residual (error) between the simulated and observed values. (Gomez & Gomez, 1984; Krause *et al.*, 2005; Willmott *et al.*, 1985)

The Efficiency Criteria (EC) used in this study were Pearson's correlation coefficient (r), coefficient of determination (R^2), Index of agreement (D), Nash-

Sutcliffe efficiency (NSE), Percent bias (PBIAS), and Root Mean Square Error (RMSE). The ECs were presented in Eqns. (26 through 30)

$$R^2 = \left(\frac{\sum_{i=1}^{n}(O_i-\bar{O})(S_i-\bar{S})}{\sqrt{\sum_{i=1}^{n}(O_i-\bar{O})^2}\sqrt{\sum_{i=1}^{n}(S_i-\bar{S})^2}} \right)^2 \quad (26)$$

$$D = 1 - \frac{\sum_{i=1}^{n}(O_i-S_i)^2}{\sum_{i=1}^{n}(|S_i-\bar{O}|+|O_i-\bar{O}|)^2} \quad (27)$$

$$\text{NSE} = 1 - \frac{\sum_{i=1}^{N}(S_i-O_i)^2}{\sum_{i=1}^{N}(S_i-O_{mean})^2} \quad (28)$$

$$PBIAS = \frac{\sum_{i=1}^{N}(O_i-S_i)\times 100}{\sum_{i=1}^{N}(O_i)} \quad (29)$$

$$\text{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(S_i - O_i)^2} \quad (30)$$

Where O, S, $\bar{S}, \bar{O}$ or O_{mean} are the observed value, simulated value, mean of the simulated value and mean of the observed value respectively.

Model Validation

The accuracy and reliability of the rainfall-induced erosion model under a broad range of operating conditions or environments were assessed by comparing the predictions from a rainfall erosion model to the measured observed field data of soil erosion. (Krause *et al*., 2005).

Different sets of soil erosion data were used for model validation.

Sensitivity Analysis

The determination of the parameters most influential on the model result is one of the steps in model development. The sensitivity measure was determined through the variation of inputs by a 20% increase in each of the building variables for the rainfall-induced erosion model. The sensitivity coefficient for each variable was derived by dividing the amount of increase or decrease in rainfall erosion by the 20% of increase or decrease in the variable. That is:

$$S.C = \frac{\Delta E}{\Delta V} \quad (31)$$

Where:

S.C is the sensitivity coefficient, ΔE and ΔV are changes in rainfall erosion and the independent variables respectively. During the analysis, one parameter was varied at a time while the other parameters were kept fixed (Bastidas *et al*., 1999; Hamby, 1994; Lee & Lee, 2006; Nguyen & Reiter, 2015).

RESULTSAND DISCUSSIONS

RESULTS

The data obtained in the study area is presented in Table 2. its shows the data used for model development and validation, it's obtained from the field for the month of April to October covering field data of Infiltration rate, Soil Erodibility, Rainfall intensity, Raindrop sizes, Hydraulic shear stress □, runoff velocity, Vegetation density, Depth of tilt, Slope length and Soil erosion. A weeks' data (cluster) is randomly selected from each month to be used for model development while the remaining were left for model validation.

Table 2: Average weekly field data

Month	Week	Infiltration (mm/hr)	Soil erodibility factor K (Kg.hr/Nm2)	Rainfall intensity (mm/hr	Raindrop sizes (mm)	Hydraulic shear stress □ (N/m2)	Runoff velocity (m/s)	Slope length (m)	Density vegetation Number /m2	Depth of tilt Dt (m)	Soil erosion Kg/m^2
April	1	128.51	0.62		1.50			80.00	0.00	0.10	
	2	125.40	0.61	60.03	1.00	0.00013	0.008	80.00	0.00	0.10	0.45
	3	130.10	0.62	92.95	0.50	0.00037	0.011	80.00	0.00	0.10	0.62
	4	131.80	0.61					80.00	0.00	0.10	
May	1	124.33	0.62	78.22	1.50	0.00038	0.013	80.00	0.00	0.10	3.76

	2	126.16	0.62					80.00	0.00	0.10	
	3	124.78	0.62	85.52	2.00	0.00039	0.015	80.00	3.60	0.10	6.20
	4	129.54	0.62	96.11	1.50	0.00420	0.014	80.00	3.80	0.10	12.70
June	1	120.55	0.62	103.15	2.00	0.00082	0.033	80.00	6.80	0.10	20.10
	2	118.93	0.62	88.52	1.50	0.00146	0.015	80.00	6.40	0.10	6.80
	3	120.10	0.62	78.94	2.00	0.00189	0.010	80.00	7.20	0.10	16.92
	4	101.79	0.61	69.50	2.00	0.00234	0.009	80.00	7.80	0.10	14.50
July	1	112.70	0.62	82.33	1.50	0.00318	0.010	80.00	8.50	0.10	16.81
	2	92.11	0.62	76.41	1.50	0.00334	0.009	80.00	4.80	0.10	12.10
	3	94.58	0.61	90.37	2.00	0.00223	0.016	80.00	4.30	0.10	4.20
	4	79.45	0.62	51.60	1.50	0.00334	0.007	80.00	6.20	0.10	8.50
August	1	73.50	0.58	34.31	1.00	0.00333	0.006	80.00	6.60	0.10	8.20
	2	55.89	0.61	53.84	1.00	0.00582	0.008	80.00	10.20	0.10	4.40
	3	52.98	0.62	44.81	0.50	0.00867	0.007	80.00	10.50	0.10	12.60
	4	76.65	0.62	69.52	1.00	0.00894	0.008	80.00	15.40	0.10	6.30
Septem ber	1	82.70	0.61	77.48	1.00	0.00857	0.009	80.00	15.10	0.10	6.50
	2	101.20	0.61	52.92	1.00	0.00877	0.007	80.00	15.60	0.10	18.80
	3	105.80	0.61	40.20	1.00	0.00887	0.007	80.00	16.60	0.10	10.20
	4	111.50	0.62	43.58	1.50	0.00779	0.006	80.00	14.20	0.10	7.20
October	1	108.33	0.60	68.55	2.00	0.00528	0.009	80.00	8.50	0.10	8.80
	2	115.35	0.58	59.40	1.50	0.00384	0.008	80.00	8.60	0.10	15.21
	3	124.84	0.58	32.22	1.50	0.00155	0.006	80.00	9.40	0.10	5.66
	4	118.26	0.58	28.75	1.50	0.00047	0.006	80.00	7.80	0.10	4.60

Determination of Component Equations

Component equations are building blocks for model equations. The parameter obtained for variables in Table 2. were substituted in Eqns 23, to 25 to obtained Table 3.

Table 3: Dimension pi terms computed

Month	Week	Π_1	Π_2	Π_3
April	3	0.0224	8533.04	0.00
May	4	0.0514	25269.96	18043.81
June	4	0.0680	16083.67	34084.29
July	1	0.0668	16639.99	39740.83
August	1	0.0197	2878.72	19717.75
September	4	0.0065	16114.53	35520.63
October	3	0.0271	21158.78	15526.72

π_{1n} was plotted against π_{2n} keeping π_{3n} constant to form the relationship as shown in Figure 2. the regression equation was used as the first component equation.

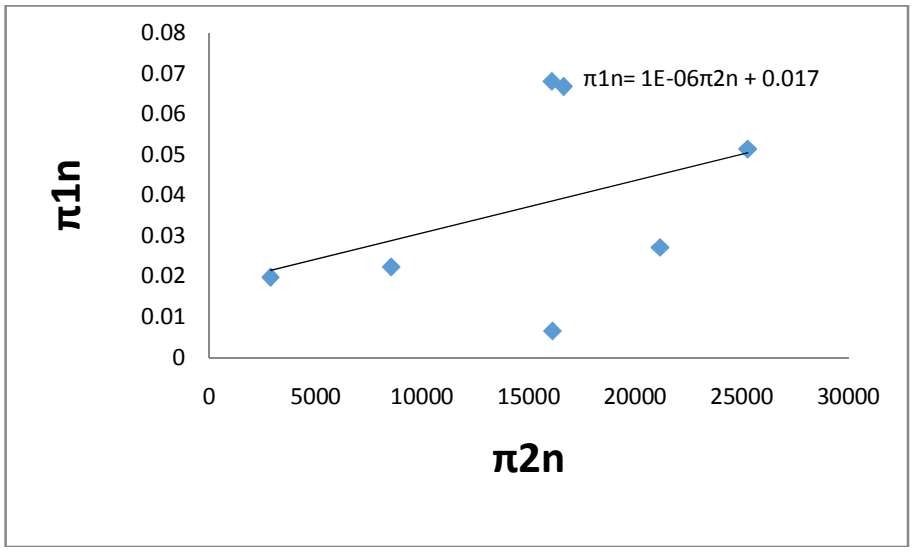

Figure 2: A plot of π_{1n}against π_{2n}(π_{3n}constant).

Similarly, π_{1n} was plotted against π_{3n} while keeping π_{2n} constant yield the relationship in Figure 3., the regression equation was used as the second component equation

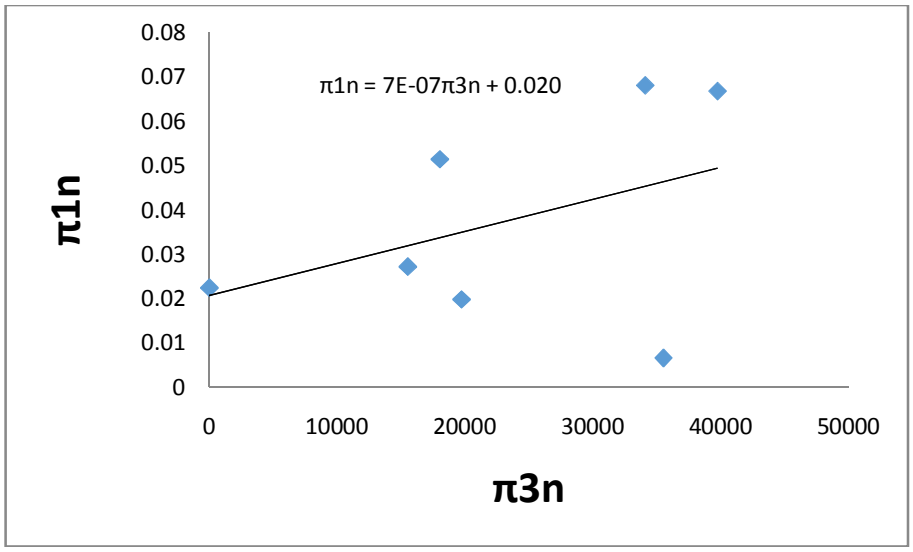

Figure 3: A plot of π_{1n}against π_{3n}(π_{2n}constant).

The two regression equations π_{1n} = 1E-06π_{2n}+0.017, and π_{1n} = 7E-07π_{3n} + 0.020, derived from the graphs were used as the component equations in the model development.

Π_{1n}= 1E-06 π_{2n} +0.017 (32)

Π_{1n} = 7E-07 π_{3n} + 0.020 (33)

The linear combinations of the two regression equations give

$\pi_{1n} = \gamma + \alpha\pi_{2n} + \beta\pi_{3n}$ (34)

(Gomez & Gomez, 1984)

The values of α, β, and γ were obtained using the multiple linear regression analysis

$$\begin{bmatrix} n & \Sigma x_1 & \Sigma x_2 \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1 x_2 \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_2^2 \end{bmatrix} \begin{bmatrix} \gamma \\ \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \Sigma y \\ \Sigma x_1 y \\ \Sigma x_2 y \end{bmatrix} \quad (35)$$

Where y, x_1, x_2 represent π_{1n}, π_{2n}, π_{3n} respectively, and n is the numbers of observations (Gomez & Gomez, 1984; Rawlings & Dickey, 1998).

The values of y, x_1, x_2 were obtained from table 2., and substituted in equation (35).

following the procedures of crammer's rule.

Using Crammer's rule procedure, γ, α, and β were found to be 0.006389, 1.28102E-06, and 2.11E-05respectively

The values of γ, α, β was substituted in equation (33) to give:

$$\pi_{1n} = \frac{\Delta_\gamma}{\Delta} + \frac{\Delta_\alpha}{\Delta}\pi_{2n} + \frac{\Delta_\beta}{\Delta}\pi_{3n} \quad (36)$$

π_{1n}= - 0.00123 -1.303× $10^{-7}\pi_{2n}$+6.8× $10^{-6}\pi_{3n}$ (37)

The Rainfall Induced Erosion Model

Where;

E is the soil erosion Kg/m^2/yr

□ is the hydraulic shear stress N/m^2

K is the soil erodibility factor Kg.hr/Nm^2

I is the soil infiltration rate mm/hr

D_t *is the* depth of tilt (m)

R_s is the raindrop size m

R_i is the rainfall intensity mm/hr

R_v is the runoff velocity m/s

S_l is the slope length m

V_d is the vegetation density Number/m^2

The parameters are obtained directly from the environment which made it easy for the user. this minimizes the error introduced in determining empirical

$$\begin{bmatrix} 7 & 105131.1657 & 3316.162 \\ 105131.1657 & 1956101576 & 57101288 \\ 3316.162 & 57101288 & 2605088 \end{bmatrix} \begin{bmatrix} \gamma \\ \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0.249491918 \\ 4384.43631 \\ 149.398 \end{bmatrix}$$

The 3×3 system of the linear equation above was solved using Microsoft excel.

Inserting the expressions of the π-terms from Eqns 32 and 33 yields the prediction equation as presented in

$$\frac{ED_tR_v}{S_l} = -0.00123 - 1.303 \times 10^{-7}\frac{I^3KR_s}{S_l} + 6.8\times 10^{-6}\frac{S_l^2R_iV_d}{I} \quad (38)$$

Equation (3.7) was further simplified to yield:

$$E = \frac{S_l}{D_tR_v}\left\{-0.00123 - 1.303 \times 10^{-7}\frac{I^3KR_s}{S_l} + 6.8\times 10^{-6}\frac{S_l^2R_iV_d}{I}\right\} \quad (39)$$

coefficients in RUSLE and WEPP (Suresh, 2002).

The model computes both sheet/rill and gully erosion while RUSLE and WEPP do not compute gully erosion (Suresh, 2002). The model can be used to compute daily, weekly, monthly soil loss as shown in Table 2. which is not obtainable in RUSLE and WEPP.

Model Validation

The result of model validation following the procedure outline shows the relationship between the observed field data and model prediction as presented in Figure 4.

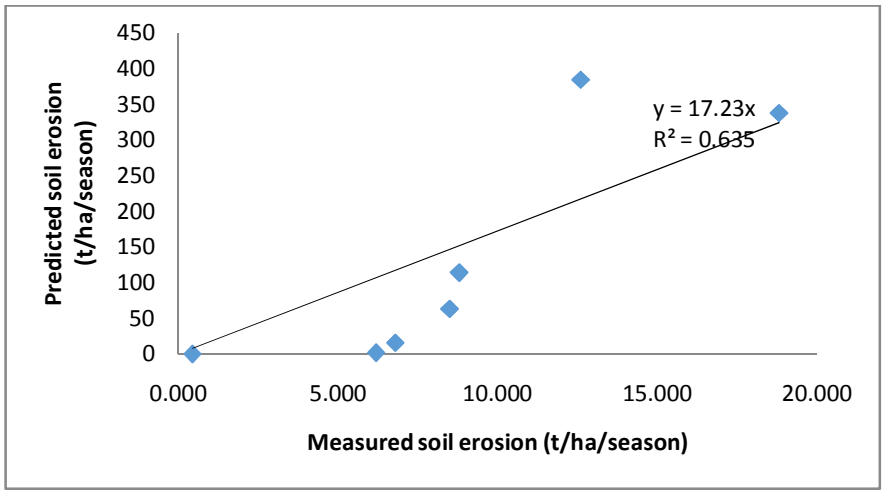

Figure 4: Comparison of predicted and measured soil erosion.

The R^2 value of 0.635 indicates a high degree of agreement between the model output and the field measured soil erosion data. This implies that the model is a good representation of real soil erosion and demonstrates the model's ability to predict rainfall-induced soil erosion.

Model Performance Evaluation

The performance of the model was evaluated by the methods outlined and the performance indices are presented in Table 4.

Table 4: Model evaluation statistics

Model evaluation statistics	Value	Optimum value
Slope	0.014	1
y-intercept	0.000	0
r	0.846	1
R^2	0.635	1
D	0.069	1
NSE	0.035	1
PBIAS	13.785	0
RMSE	0.524	0

DISCUSSION OF RESULTS

A slope of 0.014 and a y-intercept of 0.0 indicate that the model perfectly reproduces the magnitudes of measured data (Krause *et al*., 2005). Slope and y-intercept of the best-fit regression line indicate how well-simulated data match measured data.

The correlation coefficient from Table 4,. falls within the acceptable range which indicates that a strong positive linear relationship exists. The correlation coefficient, which ranges from -1 to 1, is an index of the degree of the linear relationship between observed and simulated data. When r is equal to 0 it indicates no linear relationship exists, while r is equal to 1 or -1 indicates a perfect positive or negative linear relationship exists(Krause *et al*., 2005; Willmott *et al*., 1985).

The coefficient of determination (R^2) also falls within the acceptable limit as shown in Table 4,. It describes the proportion of the variance in measured data explained by the model. It ranges from 0 to 1, with a higher value indicating less error variance, and typically value greater than 0.5 are considered acceptable.

An index of agreement (D) of 0.069 indicates a strong agreement between the measured and predicted values. It's a standardized measure of the degree of model prediction error and varies between 0 and 1. A computed value of 1 indicates a perfect agreement between the measured and predicted values, and 0 indicates no agreement at all (Willmott *et al.*, 1985).

Nash- Sutcliffe efficiency (NSE) value in Table 3.3 falls within the acceptable levels of performance. It does indicate how well the plot of observed versus simulated data fit the 1:1 line. NSE ranges between -∞ and 1.0 (1 inclusive) with NSE 1 being the optimal value. Values between 0.0 and 1.0 are generally viewed as acceptable levels of performance, whereas values ≤ 0.0 indicates that the mean observed value is a better predictor than the simulated value, which indicates unacceptable performance (Krause *et al*., 2005).

Percent bias (PBIAS) of 13.785 in Table 4., indicates model underestimation bias. It's the measure of the average tendency of the simulated data to be larger or smaller than the observed data. The optimal value of PBIAS is 0.0 with low magnitude values indicating accurate model simulation. Positive values indicate model underestimation bias, and negative values indicate model overestimation bias(Krause *et al*., 2005).

The low-value Root-mean-square error (RMSE) signified a low degree of variation between the observed and predicted values, generally, the analysis demonstrated a good predictive capacity of the model (Gomez & Gomez, 1984). Its ranges from zero (for the ideal model) to positive infinity (worst model). RMSE is biased toward peak flows.

Sensitivity Analysis

The sensitivity analysis of the model was performed following the procedure outlined. The sensitivity coefficients (S.C.) are presented in Table 5. In fact, the result in Table 5., shows that the sensitivity coefficients of the variables ranged from -119.78 to 5525.61 for all the independent variables. The result of the sensitivity analysis revealed that the model is most sensitive to changes in hydraulic shear stress that has the dominantly highest S.C. this could be attributed to its role in detachment and transportation of the soil particles (Hamby, 1994, 1995)

Table 5: Results of sensitivity analysis

Variables	Original values of variables	20 % increase in the value of the variables	Original soil erosion values	Soil erosion responses to 20 % increases in the value of the variables	Change in soil erosion	Changes in variables values	Sensitivity coefficient (S.C.)
Infiltration (mm/hr)	124.78000	149.73600	2.127	1.695	-0.432	24.95600	**-0.017**
Soil erodibility factor K	0.62000	0.74400	2.127	2.111	-0.016	0.12400	**-0.132**
Rainfall intensity (mm/hr	85.52000	102.62400	2.127	2.574	0.447	17.10400	**0.026**
Raindrop sizes (mm)	2.00000	2.40000	2.127	2.111	-0.016	0.40000	**-0.041**
Hydraulic shear stress (N/m2)	0.00039	0.00046	2.127	2.553	0.425	0.00008	**5525.610**
Runoff velocity	0.01480	0.01776	2.127	1.773	-0.355	0.00296	**-119.784**
Slope length (m)	80.00000	96.00000	2.127	3.749	1.622	16.00000	**0.101**

Density vegetation	3.60000	4.32000	2.127	2.574	0.447	0.72000	**0.621**
Depth of tilt Dt	0.10000	0.12000	2.127	1.773	-0.355	0.02000	**-17.728**

CONCLUSION

A model for estimation of Rainfall-induced erosion on sandy loam soil has been developed following the principle of the Buckingham pi theorem. The fieldwork included measurements of Hydraulic shear stress, Soil erodibility factor K, Soil infiltration rate, Depth of tilt, Raindrop size, Rainfall intensity, Runoff velocity, Slope length, Vegetation density on the experimental field plot. The component equation derived from the regression equation was used to develop the model equation. The R^2 value of 0.635 indicates a high degree of agreement between the model output and the field measured soil erosion data which implies that the model is a good representation of real soil erosion and demonstrates the model's ability to predict rainfall-induced soil erosion with a high degree of confidence. Generally, the performance evaluation of the model indicates a good predictive capacity. Also, it's signified that the model has high accuracy and precision. Likewise, the outcome of the sensitivity analysis revealed that the model is most sensitive to changes in hydraulic shear stress that has the dominantly highest S.C. this could be attributed to its role in detachment and transportation of the soil particles. The parameters in the model equation are easy to determine from the environment.

RECOMMENDATION

It is recommended for use in tropic regions because variables are easily determined which makes it better and easily adaptable. Likewise it is capable of measuring soil loss from sheet/rill and gully erosion. Similarly, the model is capable of measuring weekly, monthly and yearly soil loss of an area and it accounts for soil deposition.

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